Theoretical Models of Dynamic Network Formation

Adam Hamdan

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Abstract

A review of the theoretical literature on strategic dynamic network formation. We introduce readers to models of dynamic network formation that include myopic and farsighted agents. We then highlight two active areas of research, discussing games played on dynamic networks, and the incorporation of heterogeneity and incomplete information about the network structure. The paper concludes with some theoretical considerations for current research on dynamic social networks.

Keywords: Network Formation, Dynamic Networks, Farsightedness, Network Games, Incomplete Information.

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1 Introduction

The study of social networks has long been of interest to sociologists, economists, and computer scientists alike. From an economics perspective, the interest in networks is twofold indeed. Firstly, social networks play a critical role in shaping several types of economic behaviour and their outcomes. Examples of this can be found in informal credit-markets, the spread of political opinions, the diffusion of research and innovation, and perhaps most notably, in labour markets (see Jackson (2011) for a comprehensive overview on the influence of social networks in economics). Secondly, the nature of social networks has offered economic theorists a multitude of research questions regarding the underlying mechanisms which govern the network formation process. Accordingly, several theories of network formation have emerged in an attempt to understand why and how people form into groups, and in order to predict what network structures are likely to arise.

Within the game theoretic literature, Jackson and Wolinsky (1996) is considered to be the first paper to provide a general framework for the analysis of network formation. Largely based on the work of Aumann and Myerson (2003), the Jackson and Wolinsky model frames network formation as a static game in which every pair of agents can make cooperative costly links in exchange for a payoff. Within their framework, a value function assigns a worth to every graph g, and an allocation rule determines how the value of a given network is distributed amongst players. It is also assumed that agents get a greater payoff from direct connections than from indirect ones, according to a given rate of decay δ. Naturally, this model was quickly extended to allow for a wider range of network dynamics, including the contributions of Dutta et al. (1998), Qin (1996), Slikker and Van den Nouweland (2001), and Johnson and Gilles (2003), to name a few. Of particular interest to our paper is the introduction of time into the model, which allows agents to make and break links across several periods during the network formation process. The aim of our paper is to introduce readers to the growing body of literature on dynamic network formation, and highlight a few key areas of active research in the field.

The remainder of the paper is organized as follows. The first section covers influential models of dynamic network formation, including those with myopic and farsighted agents. We then focus on two active areas of research within dynamic networks: games played on dynamic networks, and dynamic network formation under incomplete information. The paper ends with a brief discussion on dynamic network research, touching on some theoretical and modelling considerations.

2 Strategic Dynamic Network Formation

2.1 Early Models of Dynamic Networks

The literature on dynamic network formation officially begins with Watts (2001), who extends the Jackson and Wolinsky "connections model" to a dynamic framework. In accordance with the conventional network formation setting, Watts models a social network as a graph g, where n agents, $N = \{1, 2, ..., n\}$, are represented by nodes, and their ability to directly communicate with each other is represented by edges on the graph. If players i and j are directly connected, we write $ij \in g$. In the case that every player is connected to every other player, then we have a complete graph, which we denote g^n. The model is cooperative, meaning that the links are non-directed (two-sided), and every connection must be reciprocal. Severing links, on the other hand, is a unilateral decision.

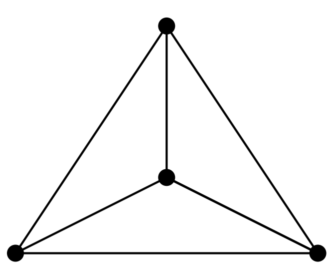
Similarly to Jackson and Wolinsky (1996), Watts assumes that agents pay to form or maintain links, and benefit from both direct and indirect connections. Intuitively, the benefit from indirect connections decreases as the distance between two agents increases. Formally, an agent i receives the following payoff $u_i(g)$ from a given network g:

$$u_i(g) = \sum_{j \neq i} \delta^{t(ij)} - \sum_{j:ij \in g} c$$

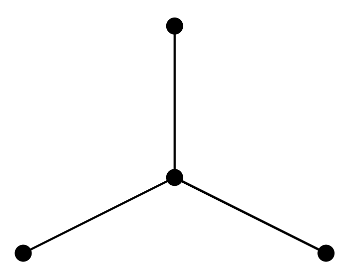
Where $t(ij)$ is the distance (number of direct links in the shortest path) connecting i and j, and $1 > \delta^{t(ij)} > 0$ represents the payoff agent i receives from being connected to j, implying that agents have more to gain from direct connections than indirect ones. For instance, if i and j are directly connected, then $\delta^{t(ij)} = \delta$. If there is no path connecting i and j, then $t(ij)$ goes to ∞ and $\delta^{t(ij)} = 0$. The cost of forming or maintaining a link is represented by the constant c. The key departure from the previous literature comes from Watts allowing players to form and sever links across time T, modelled as a countable, infinite set, $T = \{1, 2, ..., t, ...\}$. At $t = 0$, players start out with no connections (i.e. an empty graph). At each period, a pair of players ij is chosen at random and given the option to form a link, or sever any of their respectively existing links. Players are myopic, and will choose the action that maximizes their payoff at period t. This process of network formation repeats itself until no player has an incentive to make/sever any additional links.

As in most models of network formation, we are interested in knowing under which circumstances the *stability* and *efficiency* of a network is likely to be achieved. Using a fairly standard definition of efficiency, Watts defines a graph g to be efficient if it maximizes the sum of each agent's utility. Furthermore, borrowing from Jackson and Watts (2002a), a graph g is said to be *stable* if: 1) no agent i wants to form a direct link, and 2) no pair of agents ij wants to form a link or simultaneously sever any of their existing ones. This stability concept is in fact a slight extension of Jackson and Wolinsky's (1996) notion of *pairwise stability*, which does not account for the possibility of simultaneously forming and severing links.

Beyond establishing a general framework for the analysis of dynamic network formation, Watts (2001) derives several important results. She shows that if the net benefit from maintaining a direct link is greater than that of maintaining an indirect link of length 2, then the network formation process will always converge to a complete graph g^n. In this case, g^n is also the unique efficient network, and the unique stable network. In the opposite case, where the benefit from maintaining an indirect link outweighs the net benefit from maintaining a direct link, then the unique efficient and stable network is the star network. Interestingly, Watts shows that as the number of agents increases, the probability of achieving a star decreases.



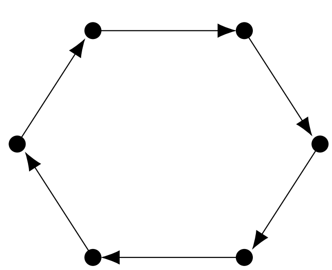
A complete graph



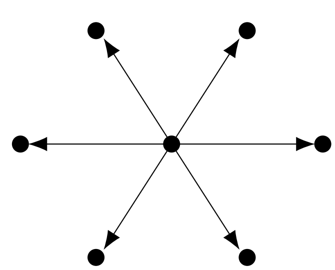
A star graph

Jackson and Watts (2002a) expand on Watts (2001) in several ways. To begin with, their model extends the dynamic network framework to a wider range of network models, such as trading networks and a matching application. They do so using the notion of an *improving path*, which they define as a sequence of networks for which a link is added or deleted at each step, to the benefit of the agents whose consent was required for the change. In the instance of trading networks, repeated *cycles* (which one can think of as a persistent sequence of network configurations) turn out to be the closest one can get to pairwise stability. Their analysis is then augmented by a stochastic element which causes the network formation process to be occasionally interrupted by exogenous errors/mutations across time. At each period t, the unintended deletion/creation of a link occurs with some small probability ϵ. Under this new lens, a graph g is *stochastically stable* if it is robust to these trembles.

The seminal paper by Bala and Goyal (2000) deals with network formation as a non-cooperative process in which agents can make directed links in order to access another agent's information, which is assumed to be *nonrival.* The paper first distinguishes between a *one-way* and *two-way* flow model, and assumes that the distance between agents does not diminish the benefit accrued from an indirect connection. In the one-way flow model, a connection will only benefit the agent who initiates the link, while the two-way flow model assumes that a link between i and j is equally beneficial to both i and j. Under the first assumption, Bala and Goyal show that if there is no decay ($\delta = 0$), then the achievable Nash networks are the empty network and the wheel network. In the case of two-way information flows, the Nash networks are the empty network and centered-sponsored star. Both results strikingly contrast Watts (2001), since the efficient network structure is almost always achieved.



A directed wheel network



A center-sponsored star

2.2 Myopia VS Foresight

Naturally, extending network formation from a static to a dynamic process raises the question of an agent's outlooks. In particular, what is the time horizon that an agent considers when making a utility-maximizing decision? As noted by Dutta and Jackson (2003), an important issue that arises when trying to model dynamic network formation is the inclusion of *farsighted* agents. As they explain, "Farsightedness would imply that players' decisions on whether to form a network are not based solely on current payoffs, but also on where they expect the process to go and possibly from emerging steady states or cycles in network formation." Indeed, allowing agents to consider the long-term repercussions of their actions can lead to drastically different network configurations. What's more, rejecting the myopia assumption seems to better align with the empirical evidence we have on network formation, which suggests that agents are hardly ever fully myopic or farsighted (see Kirchsteiger et al. (2016)).

The issue of farsightedness in network formation is first dealt with by Watts (2002), who examines the repercussions of farsighted behaviour on the ultimate topology of networks. According to Watts, forward-looking agents might be incentivized to create links in anticipation of specific network structures to emerge, even if those links do not lead to an immediate increase in the agent's payoff. Indeed, her model assumes that agents behave non-myopically, and that they discount expected future payoffs at a rate $0 \leq d \leq 1$ (note that an agent's payoff function in this model is identical to Watts (2001)). Thus, at period t, an agent will consider the discounted sum of her future payoffs:

$$u_i^t = u_i(g_t) + du_i(g_{t+1}) + d^2 u_i(g_{t+2}) + \cdots$$

Under this new assumption, Watts shows that when the cost of forming a link exceeds its immediate benefits, agents may end up forming into a circle network in which all players receive the same payoff, provided that the circle network takes form fast enough. Furthermore, as the number of agents n increases, so does the likelihood of a circle network forming.

Other papers, however, arrive at less optimistic results. In his analysis of two-way flow communication networks with farsighted agents, Deroïan (2003) finds that as the number of agents increases, efficient networks are less likely to emerge. Dutta et al. (2005) also observe that farsightedness does not always lead to efficiency, in large part due to a persistent coordination problem amongst agents. They apply minor restrictions to the worth function and allocation rule (which together are said to make a *valuation structure*) and arrive at important results concerning the existence and feasibility of efficient and stable networks. In fact, they show that for certain valuation structures, there are no equilibrium strategy profiles which yield a path towards an efficient network, while other valuation structures guarantee the existence of an achievable efficient complete graph g^n, for some equilibrium profile.

Page et al. (2005) study the implications of farsightedness within network formation in further detail. By restricting their analysis to directed links, they define a directed network to be *farsightedly stable* if it is immune to any sequence of deviations which could make an individual agent, or coalition of agents, better off. This framework also allows them to examine the relationship between the set of Nash networks and the set of farsightedly stable networks, which, in some of their examples, fail to coincide.

A similar definition is provided by Herings et al. (2009), who describe a set of networks G as *pairwise farsightedly stable* if: 1) all possible farsighted pairwise deviations to a network outside of G carry the risk of making an agent worse off or equally well off, 2) there exists a farsightedly improving path from any network g' to a network $g \in G$, and 3) G must be the smallest subset of G to satisfy conditions 1) and 2). Much like Jackson and Watts (2002a), a farsighted improving path refers to a sequence of graphs, where at each step in the sequence, a link ij is being created or severed to the strict benefit of either i, j, or both, and to the detriment of neither i nor j. Under this new definition, Herings et al. (2009) are able to show that a non-empty set of pairwise farsightedly stable networks always exists. Additionally, if a unique Pareto dominant network g exists, then the singleton $\{g\}$ is the unique pairwise farsightedly stable set.

Adopting this notion of *pairwise farsighted stability*, Grandjean et al. (2011) also arrive at several interesting observations. In cooperative symmetric networks with homogeneous agents, the stability and efficiency predictions under farsighted behaviour are akin to those achieved with myopic behaviour. This result changes when the network includes heterogeneous players (as is the case in buyer and seller networks). In these cases, farsightedness may very well sustain strongly efficient networks when myopia fails to do so entirely. Moreover, Grandjean et al. (2011)

prove that under specific valuation structures, the set of socially optimal networks and the unique pairwise farsightedly stable set are the same. This turns out to be the case when the allocation rule is componentwise egalitarian (anonymous and component additive), and the value function is top convex (there exists a strongly efficient network which maximizes per-capita value for all agents in the network).

Notice that so far, the models we've mentioned have either considered fully myopic or fully farsighted agents. By contrast, Herings et al. (2019) generalize these approaches by introducing the concept of *horizon-K farsightedness*. The definition employed for a horizon-K farsighted set is a direct extension of Herings et al.'s (2009) pairwise farsightedly stable set, except farsighted deviations are now of length K. An application to a model of criminal networks (similar to Calvo-Armengol and Zenou (2004)) is used to study the consequences of agents with heterogeneous time horizons. Likewise, Luo et al. (2021) study network formation in the presence of both myopic and farsighted agents. Building on the work of Herings et al. (2019), they establish the similar notion of a *myopic-farsighted stable set*, which essentially combines the requirements of pairwise myopic stability, from Jackson and Watts (2002a), and pairwise farsighted stability, from Herings et al. (2009).

In an effort to reconcile empirical evidence on network formation with theoretical predictions, the paper by Song and Van Der Schaar (2020) constructs yet another approach to foresight. In their model, farsighted agents are able to learn enough from the network formation history while facing a uniform punishment mechanism which causes them to arrive at the efficient network structure. We assume that at each period, agents observe a *public signal*, generated by a signal device y which conveys information about the network formation history up to period t. Inspired by the literature on repeated games, the signalling device lets agents know whether to engage in a period of Cooperation or Punishment: If an agent observes that other agents have departed from the cooperation phase, then they enter into a punishment phase which permits for cooperation to be restored until the efficient structure is achieved. Under this constructed monitoring structure, a *network convergence theorem* is derived which says that if agents are patient enough, then there exists an equilibrium which converges to a strongly efficient network g. Surprisingly, agents need not know the entire network formation history nor *who* deviated from the co-operative phase in order for efficiency to be achieved. As Song and Van Der Schaar show, the convergence theorem even holds when agents are restricted to *Markov* *strategies* in which they only consider network formation at time $t-1$ when making a decision at time t.

3 Topics in Dynamic Network Formation

3.1 Dynamic Network Games

In many cases, an individual's actions heavily depend on her position in a network as well as the actions taken by her neighbours. To illustrate this, consider the scenario studied by Bramoullé et Kranton (2007), where a local non-excludable public good is provided by agents in a network. Since an agent's contribution will also benefit all of her direct neighbours, every agent has a clear incentive to free-ride on her neighbour's contribution. Network behaviours of this type have also been documented in settings of technology adoption and peer effects. Motivated by the strategic implications of these contexts, a rich literature on network games has emerged (see Galeotti et al. (2010), and Jackson and Zenou (2015) for a detailed account of network games). Within our scope of focus, we consider a subset of this literature that studies games played on dynamically evolving networks.

A network game represents a situation in which a player's payoff is conditional on the actions taken by her neighbours. In the case of games on dynamic networks, an agent engages in both network formation and strategic play. A particular example of this is studied in Jackson and Watts (2002b), who consider social coordination games where the choice of partners is endogenous. As usual, a network made up of n myopic agents is represented by a graph g. Links are assumed to be costly and cooperative. Amongst every pair of connected agents i and j, a coordination game is played, the outcome of which determines the payoff received. Hence, each agent maximizes her payoff function $u_i(g^t, a^t)$, where g^t is the network configuration and a^t is the payoff from the coordination game. At each period, a randomly chosen pair of agents decides on link creation/deletion before connected agents play. Following Jackson and Watts (2002a), all actions are interrupted by random trembles and stochastic stability is used to identify equilibrium networks. The main insight of the paper is that network endogeneity enables coordination. The intuition for this is simple. Since links are costly and endogenous, an agent i will choose to sever her connection with agent j if it is the case that i and j perpetually fail to coordinate.

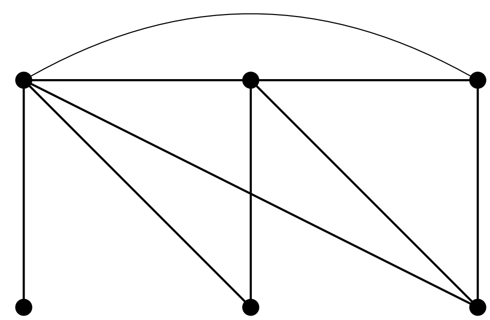
An analogous setup is used by Goyal and Vega-Redondo (2005). Their model explores the influence of partner choice on social coordination games when social links are one-sided. Within the coordination game, two pure strategy equilibria are identified, one leading to the efficient outcome, the other leading to the risk-dominant outcome. It is shown that if linking costs are low, then individuals coordinate on the risk dominant action, and if linking costs are high, then the socially efficient action is the prevailing strategy. In the long run, stochastically stable networks are almost always complete graphs.

Differing results are found when an anti-coordination game is played amongst connected agents. Assuming that links are one-sided and costly, Bramoullé et al. (2004) find that as the cost of link formation decreases, the complete network becomes the only stable configuration. As linking costs increase, a wider range of stable architectures becomes admissible, including bipartite and semi-bipartite networks. In both cases however, networks are neither unique nor efficient.

König et al. (2009) also examine the relationship between dynamic network formation and strategic behaviour. Their approach extends the framework introduced by Ballester et al. (2006) to dynamic networks, utilizing a two-stage game. In the first stage, agents simultaneously choose an effort level under an exogenously fixed network structure. Following Ballester et al. (2006), the game played involves local complementarities, and agents with the highest levels of *Bonacich-centrality* exert the highest effort and receive the highest payoff. Borrowed from the sociology literature, the level of Bonacich-centrality can be interpreted as a measure of an agent's influence/network centrality, and is equal to:

$$b_i(G,\lambda) = \sum_{k=0} \lambda^k - \sum_{j=1} (\mathbf{A}^k)_i j$$

Where λ is the decay factor, and $\sum_{j=1} (\mathbf{A}^k)_i j$ is the number of walks of length k in G starting from i. In the second stage of the game, a process of network formation begins. At each period t, an agent i is randomly chosen and decides with whom she wishes to form a link by picking a best response to the current network. Her knowledge of the network is local (she can only form links with friends of friends), and she chooses to connect with the agent j that increases her Bonacich-centrality the most. If i has no neighbours at $t = 0$, she picks the best link among all the agents. Ultimately, the two-stage game produces stationary nested-split graphs.



A nested split-graph

In a later publication, König et al. (2014) refine this model and test it empirically. Under stochastic stability, the network formation process also converges to nested split graphs, matching with their observational data.

Finally, Gauer and Hellman (2017) study network formation in tandem with bargaining games. Using Manea's (2011) approach to bargaining in networks, they examine the effect of network formation on the bargaining process, and vice versa. Indeed, a stage of dynamic network formation precedes the bargaining game, so that agents, although homogeneous in type, now have heterogeneous bargaining positions. Gauer and Hellman find that the set of generically pairwise stable networks can take several shapes, some of which might include an isolated agent. Among non-isolated players however, the bargaining outcome turns out to be equitable: players might have incurred different linking costs, but they all share the same payoff.

3.2 Incomplete Information

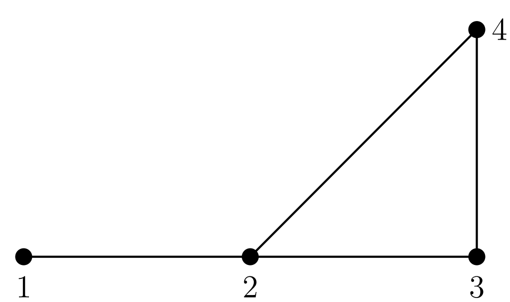
All of the models we have discussed thus far have either implicitly or explicitly assumed that agents are perfectly informed with regards to the network structure and the payoff there is to receive from forming a given link. A much more natural approach however, is to assume that agents start out with some initial *belief* about the structure of the network they are participating in, and subsequently update this belief as the network formation process unravels.

As Song and Van der Schaar (2015) illustrate, the assumption of incomplete information has a considerable effect on the dynamic evolution of networks and their ultimate shape. Their model begins with an identical setup to that of Watts (2001), with the only difference being that agents have no initial information regarding the network structure. Agents behave myopically and are heterogeneous in type, where every agent's type k is drawn from a finite set of types X according to a distribution H. Since the benefit from linking to a single agent depends on that agent's type, the individual's payoff function is now replaced by an expected benefit function, which depends on the prior type distribution H. At the initial period $t = 0$, agents only know their individual type as well as the type distribution H of other players in their network. Accordingly, an agent i chooses to connect with an agent j based on her belief of j's type, which she can only know for certain once i and j are connected. In other words, agents begin with some common prior beliefs about the likelihood of interacting with specific types of agents, and engage in a process of *learning* as network formation occurs. Song and Van der Schaar show that if the expected value of forming a link is smaller than the cost c that agents face, then an agent's beliefs will never be updated and will remain forever at the prior. If the opposite is true, then as time advances and agents become increasingly connected, complete information is eventually achieved in any stable network. The authors also show that under incomplete information, a wider number of stable network configurations can emerge. For example, lower-value agents might secure a high degree of connectivity in early stages of the game out of sheer luck, a result which directly contradicts the previous literature on agent heterogeneity in network formation (see Galeotti (2006), and Galeotti et al. (2006)).

Zhang and Van der Schaar (2015) consider the impact of incomplete information in a slightly different way. In their model, each agent has a specific quality, and gets to learn about other agents' qualities over time. Much like our previous model, agents begin with a knowledge of their own type as well as the distribution of types $N(\mu_i, \sigma_i^2)$. Unlike Song and Van der Schaar (2015) however, agents learn and make linking decision simultaneously, and the learning process is a gradual one. For each link connecting i to other agents j, a noisy benefit signal is sent to players j indicating the quality of i with a certain degree of precision. According to the equation for quality diffusion, as an agent's number of direct neighbours increases, its volatility rate decreases and her true quality q_i is more readily learned. Zhang and Van der Schaar (2015) show that agents of high quality are more prone to developing a good reputation and a high degree of connectivity, while those with low initial quality are likely to get a bad reputation and eventually become ostracized.

In a later paper, Zhang and Van der Schaar (2017) develop a framework that allows to jointly study the effects of homophily, incomplete information, and learning on dynamic network formation. Agents behave similarly to Zhang and Van der Schaar (2015), but now exhibit *homophily*, that is, a desire to associate with agents of similar characteristics. In particular, an agent i will only form a link with j if she believes they are of the same type, and will only maintain a link with j if the prior turns out to be true. Players' beliefs evolve, and links are created until a final network topology is achieved. It is shown that under these assumptions, incomplete information leads to lower levels of clustering in equilibrium.

Recent work by Song and Van der Schaar (2018) introduces a unifying model capable of analyzing the interplay between network games, dynamic network formation, and incomplete information of the network structure. The framework restricts itself to *connected networks*, in which every two agents are either directly or indirectly connected. Formally, a network g is *connected*, if for every two agents i and j, there exists a path $ik_1, k_1k_2, ..., k_n, j \in g$ linking i and j together. In a preliminary stage, agents simultaneously make cooperative costly links with one another. At the end of this period, players are also informed of the formation history through a signalling device, or monitoring structure $y : \phi \to Y$, where ϕ and Y are the set of all possible formation histories and the set of signal realizations, respectively. In the second stage, different games are played in every non-empty complete sub-network of our network. For example, in the network g = {12,23,24,34} illustrated below, the non-empty complete sub-graphs are {12}, {23}, {24}, {34}, and {23,24,34}. In total, five games will be played in g. In addition to learning about the network formation history, an agent retains private knowledge of every action taken by every agent in every game she played.



A network with links {12,23,24,34}

Hence, the *public* information learnt through the monitoring structure along with the *private* information acquired at each period of strategic play cause network formation and network games to affect each other's outcomes. In most cases, games result in a higher level of social welfare when the network is endogenous.

4 Discussion

We have covered some fundamental models of dynamic network formation and discussed two branching topics within the literature. While research on dynamic networks has greatly expanded in the past few years, there are a number of considerations that remain to be addressed.

To the best of our knowledge, not much work has been done on the issue of farsighted behaviour when agents have incomplete information about other players' types. It would also be interesting to analyze how different updating rules affect the ultimate network structure. For instance, one could assume that agents start out with no belief about the underlying distribution of types H, and instead use their experience to make up a belief, as the network formation stages unfold. Upon engaging with a "bad" type for example, an agent is more reluctant to form future links as she becomes increasingly pessimistic about the underlying distribution H. In such a setting, an agent's first few encounters would have a lasting effect on their behaviour and willingness to form links, possibly affecting the final network structure. As pointed out by Song and Van der Schaar (2018), future research could also try to incorporate a language/communication system and study its consequences on a player's behaviour. Another interesting avenue could be to expand upon an agent's set of available actions, potentially allowing agents to change their location in spatial network models like Johnson and Gilles (2003) or change their group in insider-outsider models like Galeotti et al. (2006).

The permeating effects of social networks on economic behaviour warrant a thorough theoretical understanding of network formation. Given the relative recency of the field, and the increased availability of high quality network data, one can expect dynamic network formation to remain a promising line of research for empirical and theoretical economists alike.

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